Calculation of Unsteady Transonic Aerodynamics for Oscillating Wings with Thickness

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An analytical approach is presented to account for some of the nonlinear characteristics of the transonic flow equation for finite thickness wings undergoing harmonic oscillation at sonic flight speed in an inviscid, shock-free fluid. The thickness effect is accounted for in the analysis through use of the steady local Mach number distribution over the wing at its mean position by employing the local linearization concept and a coordinate transformation. Computed results are compared with that of the linearized theory and experiments. Application to a flutter problem is shown.

Nomenclature

A = aspect ratio
a = pitch axis (measured aft from most forward point on wing)
B = $B(x,y,z,t) = 0$, body surface position
b = reference length—wing centerline chord, ft
c = local speed of sound (normalized to U_∞)
f_j = jth mode shape
Im = "imaginary part of"
k = reduced frequency, $k = \omega b / U_\infty$
L_{ij} = generalized aerodynamic force coefficient
M = local Mach number
N = number of boxes along centerline chord
O = "order of"
S = wing planform area
s = local semispan
t = dimensionless time (normalized to b/U_∞)
U_∞ = freestream velocity, fps
V = flutter speed, fps
W = wake
w = downwash
x,y,z = dimensionless Cartesian coordinates
δ_i = ith mode amplitude of motion
σ = semispan at the wing trailing edge
Φ = small perturbation velocity potential, $\Phi = \phi + \varphi$
ϕ = steady part of Φ
φ = unsteady part of Φ
ω = angular velocity, rad/sec
τ = thickness ratio
θ_{ij} = phase angle of the generalized aerodynamic force coefficient L_{ij}, degrees
$(\ )_{x,xt}$ = partial derivatives
$(\ )_o$ = magnitude of the oscillatory quantity
$(\ )_d$ = doublet
$(\ )_{i,j}$ = (i,j)th box
$(\ )_\infty$ = freestream condition
$(\sim)$ = transformed quantity

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Index categories: Nonsteady Aerodynamics; Subsonic and Transonic Flow; Aeroelasticity and Hydroelasticity.

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I. Introduction

THE study of flutter and other aeroelastic responses of an aircraft requires adequate knowledge of the forces acting on three-dimensional wings in oscillatory motion. Such aeroelastic problems are frequently critical in the transonic speed range. The physical problem is governed by a nonlinear partial differential equation with nonlinear boundary conditions, for which an exact solution is not known to exist. The basic small perturbation equation governing the velocity potential for transonic flow over a thin wing or a slender body is well known.[1,2] However, the nonlinearity which remains present prevents closed-form solutions from being obtained except in a few special cases.[1] For a low-amplitude, high-frequency oscillation where the unsteady part is considered to be a small disturbance to the steady part, the steady-state properties can be completely uncoupled from the unsteady equation, and the governing equation for unsteady transonic flow can be linearized.[2,3] Almost all unsteady transonic flow theoretical work lies within the framework of linearized theory where the thickness effect of the wing is neglected.

More recent studies[4] indicate that wing thickness, which enters into the mathematical nonlinearity, can significantly affect unsteady pressures in some speed ranges. Moreover, oscillatory transonic aeroelastic instability (i.e., flutter) often occurs at frequencies below the range of validity of the transonic linearized theory. The object of the work, therefore, is to develop an approximate method for accounting for the more important effects of finite wing thickness in order to predict transonic oscillatory aerodynamic parameters at frequencies lower than those for which linearization is valid. The present study is limited to attached, shock-free flow.

An important consequence of the previously mentioned linearization is the suppression of deviations in local Mach number from freestream value. Since these deviations have appreciable effect on propagation of pressure disturbances over the lifting surface, significant improvement in the theory may be accomplished by "recoupling" the steady and unsteady flow parameters so that solutions may approximately consider variations in mean local Mach number caused by finite wing thickness. In the present development, this is achieved by considering all of

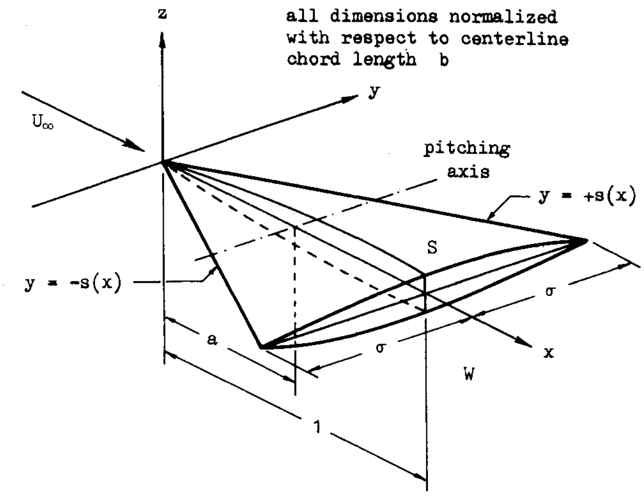

Fig. 1 Physical coordinates and sample wing geometry.

the steady-flow parameters over the wing to be invariant within a small finite region. This latter assumption, equivalent to the concept of local linearization, permits the nonlinear differential equation for the velocity potential to be reduced to a linear equation with variable coefficients containing the local Mach number. By means of an appropriate coordinate transformation, the equation becomes identical with the usual linearized transonic unsteady-flow equation with constant coefficients. Numerical results are then obtained for wings with finite thickness by means of the well-known sonic-box (linearized theory) method[5,6] applied to the wing in the transformed space.

Since this investigation is exploratory, the limitations on wing planform geometry imposed by the sonic-box method of Ref. 5 have not been removed. The types of wings which can be treated are those having unswept trailing edges, without control surfaces. Also, because of the transformation method used, the mean steady flow everywhere over the wing must not be very different from that of the undisturbed stream.

Calculations using the present method were made to evaluate the thickness contribution to the unsteady aerodynamic forces by comparison with cases not having any thickness effect. The wings considered were a) rectangular wings with a bicircular-arc profile, and b) delta wings with an elliptic lateral cross section. Comparisons are made with the sonic-box method, for cases without thickness effects, as well as with available test data on finite thickness wings. The method is also used to predict the effects of wing thickness on the flutter characteristics of a 45° delta wing at $M_\infty = 1.0$.

II. Problem Formulation

A. Basic Equation

Consider a wing performing a small-amplitude oscillation around its zero angle-of-attack position in a steady transonic flowfield. The wing is assumed to be smooth and thin enough so that the small-perturbation velocity potential equation for transonic flow can be applied. The physical coordinates and a sample wing geometry are shown in Fig. 1.

The basic equation containing only the linear terms of the small perturbation velocity potential is

$$\left(1 - \frac{1}{c^2}\right)\Phi_{xx} + \Phi_{yy} + \Phi_{zz} - \frac{1}{c^2}(2\Phi_{xt} + \Phi_{tt}) = 0 \quad (1)$$

This may be considered as a linear equation with variable coefficients, and all quantities in Eq. (1) are dimensionless. If the nonconstant coefficient in Eq. (1) is approximated by using

$$1/c^2 \cong M^2(1 - 2\Phi_x)$$

and only the terms linear in Φ are retained, the following results:

$$(1 - M^2)\Phi_{xx} + \Phi_{yy} + \Phi_{zz} - M^2(2\Phi_{xt} + \Phi_{tt}) = 0 \quad (2)$$

where M is the local Mach number. This equation maintains the mixed-flow property usually observed in the transonic flow regime. It is not, however, in the form commonly used in transonic flow studies because of the presence of a nonconstant coefficient.

If the mean steady local Mach number M is assumed to be independent of the unsteady part, Eq. (2) can be written as

$$(1 - M^2)\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \quad (3)$$

and

$$(1 - M^2)\varphi_{xx} + \varphi_{yy} + \varphi_{zz} - M^2(2\varphi_{xt} + \varphi_{tt}) = 0 \quad (4)$$

where Eqs. (3) and (4) are, respectively, for steady and unsteady flow. In this study, it is assumed that the solution of Eq. (3) is available; thus, the local Mach number representing the thickness effect in Eq. (4) is treated as a known quantity.

In transonic flow, $1 - M_\infty = 0(\phi_x)$; and the assumption that $k \gg |1 - M|$ allows the linearization of the unsteady equation (4). Consistent with this reasoning leading to the linearized theory especially for $M_\infty = 1$, it is possible to neglect the first term in comparison with the other terms in Eq. (4) for those wings over which the local Mach number is not very different from that of the undisturbed flow. Thus, Eq. (4) for $M_\infty = 1$ becomes

$$\varphi_{yy} + \varphi_{zz} - M^2(2\varphi_{xt} + \varphi_{tt}) = 0 \quad (5)$$

This equation may be considered as a linear equation with variable coefficients. For harmonic motion, the small perturbation φ may be written as

$$\varphi = \varphi_o e^{ikt}$$

and Eq. (5) becomes

$$\varphi_{o,yy} + \varphi_{o,zz} - M^2(2ik\varphi_{o,x} - k^2\varphi_o) = 0 \quad (6)$$

B. Boundary Conditions

The boundary conditions to be satisfied by Eq. (5) or (6) are that the disturbance must vanish at infinity, and the flow must always be tangent to the wing surface. The former will be satisfied by the type of solution chosen and the latter is expressed as[7]

$$DB/Dt = 0$$

where $B(x,y,z,t) = 0$ defines the body surface. For a very thin wing, the tangency condition can be linearized; and the unsteady part is

$$\varphi_z = h_x + h_t \text{ on } z = 0 \quad (7)$$

Use of the linearized tangency condition is justified for the case of thin wings performing small-amplitude oscillations. Finally, for harmonic motion, Eq. (7) can be written as

$$\varphi_{o,z} = h_{o,x} + ikh_o \text{ on } z = 0 \quad (8)$$

III. Local Linearization

A. Concept

The actual physical problem is governed by nonlinear partial differential equations with nonlinear boundary conditions, and defies exact solution. Hence, we confine ourselves here to an adaptation of the linear theory to account for the effect of wing thickness insofar as it produces a nonuniform mean flow, including possibly a local supersonic region without a terminating shock.

One technique which permits a linear method is to account for the nonlinearity in an approximate fashion, based on the concept of "local linearization." The underlying assumption is that the physical state, usually governed by nonlinear equations, is adequately described within a limited region by related linear equations in which all parameters involved have their local values taken to be invariant. This involves replacement of the nonlinear equations with linear equations having variable coefficients.

These methods were introduced into aerodynamics in an intuitive way for steady flow by Spreiter and Alksne,[8] and have now been rigorously validated by means of the method of parameteric differentiation by Rubbert and Landahl.[9] This approach suggests that, in the case of unsteady flow, the calculations can be carried out with sufficient accuracy, using the linearized equations which contain the local values of the steady-flow parameters. In his linearized theory, Landahl[10] cites evidence for the validity of applying the concept of local linearization to the case of unsteady flow.

Moreover, in the case of supersonic flow, it was pointed out by Ashley[11] that a simple way of potentially improving the accuracy of unsteady flow calculations is to use the linearized velocity potential equation, but with the Mach number of the undisturbed flow replaced by the local Mach number which varies spatially due to thickness, mean angle of attack, and/or camberline shape. This work has been extended in two ways. Sankaranarayanan and Vijayavittal[12] applied Ashley's[11] approach to supersonic flow past delta wings and found that the general effect of thickness is to reduce the flutter speed; meanwhile, Kacprzynski[13] examined the three-dimensional effects more fully. The above mentioned investigations are concerned with thickness effects in supersonic flow. Except for the work of Albano and Andrew[14] relatively little information is available about the behavior of an oscillating finite wing with thickness in the transonic flow regime.

B. Procedure

Since the present analysis is exploratory, only wings with unswept trailing edges, zero mean angle of attack, and zero camber are considered. These, however, are not essential limitations.

By introducing the following modified Prandtl-Glauert transformation

$$\left.\begin{aligned}\tilde{x} &= x\\ \tilde{y} &= M(x,y)\times y\\ \tilde{z} &= M(x,y)\times z\end{aligned}\right\} \tag{9}$$

into Eq. (6) and neglecting the spatial derivatives of M (here, M is considered as a parameter), the following is obtained:

$$\tilde{\varphi}_{o,\tilde{y}\tilde{y}} + \tilde{\varphi}_{o,\tilde{z}\tilde{z}} - 2ik\tilde{\varphi}_{o,\tilde{x}} + k^2\tilde{\varphi}_o = 0 \tag{10}$$

where

$$\tilde{\varphi}_o(\tilde{x},\tilde{y},\tilde{z}) = M(x,y)\times\varphi_o(x,y,z) \tag{11}$$

The corresponding tangency condition becomes

$$\tilde{\varphi}_{o,\tilde{z}} = \tilde{h}_{o,x} + ik\tilde{h}_o \tag{12}$$

where

$$\tilde{h}_o = h_o$$

Equations (10) and (12) are in the forms that Rodemich and Andrew[5] used in their sonic-box method. Thus a finite-thickness wing transformed according to Eq. (9) may be treated by the sonic-box method in the transformed space.

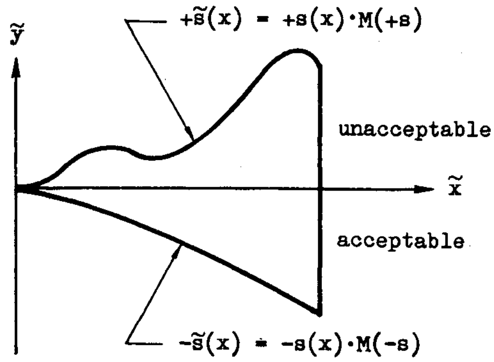

Fig. 2 Transformation of leading edge.

Even though the local Mach number may be greater than or less than unity on a given wing, the implication of the procedure outlined previously is that the locally subsonic or supersonic character of the flow is not the predominant influence on the oscillatory potential. Instead, upstream propagation of disturbances in local subsonic regions, for example, is implied to be of secondary importance and is not represented. The local Mach number is treated as a parameter, with local disturbances unable to travel upstream by virtue of the Mach freeze.

C. Limitations of the Proposed Transformation

The sonic-box method developed by Rodemich and Andrew[5] is restricted to a wing with no part of it downstream of any part of its wake(s). That is, the slope of the leading edge of the wing $d\tilde{s}/d\tilde{x}$ is not allowed to change signs as shown in Fig. 2. The leading edge shown in the positive $\tilde{y}$ quadrant is unacceptable because its slope changes sign between the apex and the trailing edge of the wing. Conversely, the slope of the leading edge shown in the negative $\tilde{y}$ quadrant does not change sign, thus it is acceptable.

Since the sonic-box method is applied to the transformed wing in the present study, this restriction may be expressed as

$$(d/d\tilde{x})[\pm\tilde{s}(\tilde{x})] \geq 0$$

or written in the quantities for the physical plane

$$\pm s(x) = -M(\pm s)\times\frac{(d/dx)[\pm s(x)]}{(d/dx)[M(\pm s)]} \tag{13}$$

where $M(\pm s)$ represents the local Mach number along the leading edges.

In addition to the limitation shown in Eq. (13), the coordinate transformation employed herein imposes another limitation concerning the spanwise Mach number distribution. That is, at x = const

$$\frac{d\tilde{y}}{dy} > 0 \text{ for } 0 < |y| < s$$

Figure 3 illustrates the nature of the acceptable and the unacceptable functional forms. In the sketch, the relation $\tilde{y} = y\cdot M(x,y)$ is drawn as a solid line. As can be seen in the illustration, the transformed local span in $\tilde{y} \geq 0$ is single valued (acceptable) whereas that in $\tilde{y} < 0$ is multivalued (unacceptable). The multivalued transformation creates a type of foldover wing in the transformed space which does not correspond to physical reality and to which the sonic-box method does not directly apply. This condition may be relaxed somewhat if at those points where the values of $d\tilde{y}/dy = 0$ occur within the range $|\tilde{y}|$

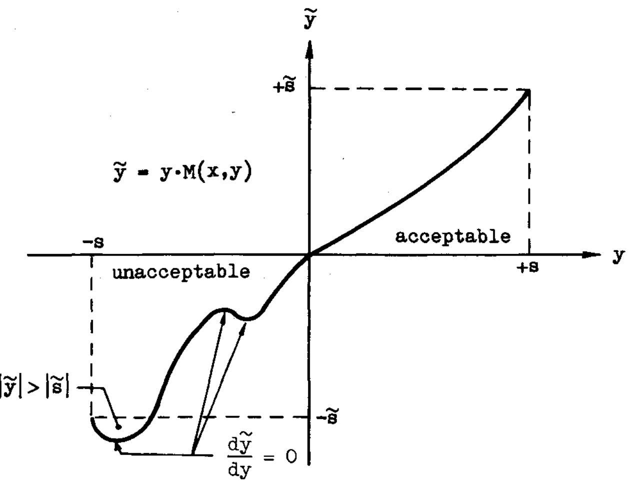

Fig. 3 Transformation of spanwise coordinate.

$< |\tilde s|$, $\tilde y$ only deviates slightly from the required monotonically increasing nature as y is transformed into $\tilde y$ from 0 to s.

IV. Method of Solution

A. Basic Solution

Since the basic equation, Eq. (6), and the tangency condition, Eq. (8), to be satisfied in the transformed space are identical to those used in Ref. 5, the calculation may be carried out in the transformed space using a computer program modified from the one developed by Rodemich and Andrew.[5]

The basic solution satisfying Eq. (6) and the condition at infinity for a thin wing having its mean position lying in the $\tilde x\tilde y$ plane, representing a point doublet with its axis oriented in the $\tilde z$ direction, is

$$\tilde\varphi_d(\tilde x,\tilde y,\tilde z) = \begin{cases}0, & \tilde x\le 0\\ \dfrac{ik}{2\pi}\dfrac{\tilde z}{\tilde x^2}\times\exp\left[-\dfrac{ik}{2}\left(\tilde x+\dfrac{\tilde y^2+\tilde z^2}{\tilde x}\right)\right], & \tilde x>0\end{cases} \qquad (14)$$

Then the solution to Eq. (6) for a distribution of doublets can be written as

$$\tilde\varphi_o(\tilde x,\tilde y,\tilde z) = \begin{cases}0, & \tilde x\le 0\\ \displaystyle\iint_{\tilde x-\xi>0}\rho(\xi,\eta)\times\tilde\varphi_d(\tilde x-\xi,\tilde y-\eta,\tilde z)d\xi d\eta, & \tilde x>0\end{cases} \qquad (15)$$

where $\rho(\xi,\eta)$, representing the doublet strength, may be any function such that the integral exists.

The downwash at a point $(\tilde x,\tilde y)$ on $\tilde z = 0^+$ of the wing mean plane, may be obtained from Eq. (15) by differentiating both sides with respect to z. Thus

$$\tilde w(\tilde x,\tilde y,0^+) = \iint_{\tilde W+\tilde S}\rho(\xi,\eta)\times\psi(\tilde x-\xi,\tilde y-\eta,0^+)d\xi\, d\eta \qquad (16)$$

where

$$\psi(\tilde x,\tilde y,0^+) = \lim_{\tilde z\to 0^+}\frac{\partial}{\partial\tilde z}[\tilde\varphi_d(\tilde x,\tilde y,\tilde z)] = \begin{cases}0, & \tilde x\le 0\\ \dfrac{ik}{2\pi}\dfrac{1}{\tilde x^2}\times\exp\left[-\dfrac{1}{2}ik\left(\tilde x+\dfrac{\tilde y^2}{\tilde x}\right)\right], & \tilde x>0\end{cases}$$

$$\rho(\tilde x,\tilde y) = \tilde\varphi_o(\tilde x,\tilde y,0^+)$$

and $\tilde W+\tilde S$ represents the portion of the transformed wing plus wake for which $\tilde x-\xi>0$. Thus, the boundary-value problem becomes

$$\tilde w(\tilde x,\tilde y,0^+) = \iint\tilde\varphi_o(\xi,\eta,0^+)\times\psi(\tilde x-\xi,\tilde y-\eta)d\xi\, d\eta \quad \text{for } (\tilde x,\tilde y)\text{in } \tilde S \qquad (17)$$

and

$$\left(\frac{\partial}{\partial\tilde x}+ik\right)\tilde\varphi_o(\tilde x,\tilde y,0^+) = 0,\ \text{for } (\tilde x,\tilde y)\text{ in }\tilde W. \qquad (18)$$

B. Doublet Box Method

The amplitude of oscillation of the thin wing under consideration is assumed to be small. The mean position of the wing is considered to lie approximately in the $\tilde x\tilde y$-plane with its nose at the origin and with an unswept trailing edge. The freestream is assumed to be parallel to the $\tilde x$ axis and at a Mach number of unity. The value of the unsteady potential on the wing may be found by using Eq. (17).

To get an approximate solution of Eq. (17), let the $\tilde x\tilde y$ plane be covered with a grid of square boxes with the box edges parallel to the $\tilde x$ and $\tilde y$ axes. Let the region E be composed of all boxes whose centers lie in $\tilde S$, the transformed wing planform surface. Thus, E is an approximation to $\tilde S$ by boxes. The potential $\tilde\varphi_0$ over each box is assumed to be constant, that is, $\tilde\varphi_{0,ij}$ = const over (i,j)th box E_{ij}. The tangency condition is applied to the center $(\tilde x_i,\tilde y_j)$ of each box E_{ij} in E, and the region of integration is replaced by E. Thus, Eq (17) gives a system of linear algebraic equations for the $\tilde\varphi_{0,ij}$'s, which can be expressed in the form

$$\sum_{j'}A(0,|j-j'|)\tilde\varphi_{o,ij'} = \tilde w(\tilde x_i,\tilde y_j) - \sum_{i'<i}\sum_{j'}A(i-i',|j-j'|)\tilde\varphi_{o,i'j'} \qquad (19)$$

where

$$A(i-i',|j-j'|) = \iint_{E_{i'j'}}\psi(\tilde x_i-\xi,\tilde y_j-\eta)d\xi\, d\eta \qquad (20)$$

If the wing is symmetric about the $\tilde x$ axis, then only modes of oscillation that are symmetric or antisymmetric in y need to be treated. Thus one only needs to consider one-half of the wing in a computation. For symmetric modes, Eq. (19) becomes

$$\sum_{j'\ge 1}[A(0,|j-j'|)+A(0,j+j'-1)]\tilde\varphi_{o,ij'} = \tilde w(\tilde x_i,\tilde y_j) - \sum_{i'<i}\sum_{j'\ge 1}[A(i-i',|j-j'|)+A(i-i',|j+j'-1|)]\tilde\varphi_{o,i'j'} \quad \text{for } j\ge 1 \qquad (21)$$

The equations for $j\le 0$ are implied by $j\ge 1$.

For antisymmetric modes, Eq. (21) applies, with the sums of values of A replaced by differences.

C. Calculation Procedure

A major part of the computer program used in this study is adapted from that developed by Rodemich and Andrew.[5] The wing with thickness is transformed using the relations in Eq. (19). The equation to be solved and the boundary condition to be satisfied in the transformed space become those in Eqs. (10) and (12), respectively. Then the sonic-box linearized-theory method of Ref. 5 is applied directly in the transformed space.

The leading edge of the planforms of both the physical and transformed wings must not have any local maximum in the spanwise direction within the range $0\le x\le 1$; that is, the local semispan must not decrease as x increases from the nose to the trailing edge. For computational convenience, the leading edge of any given wing planform is

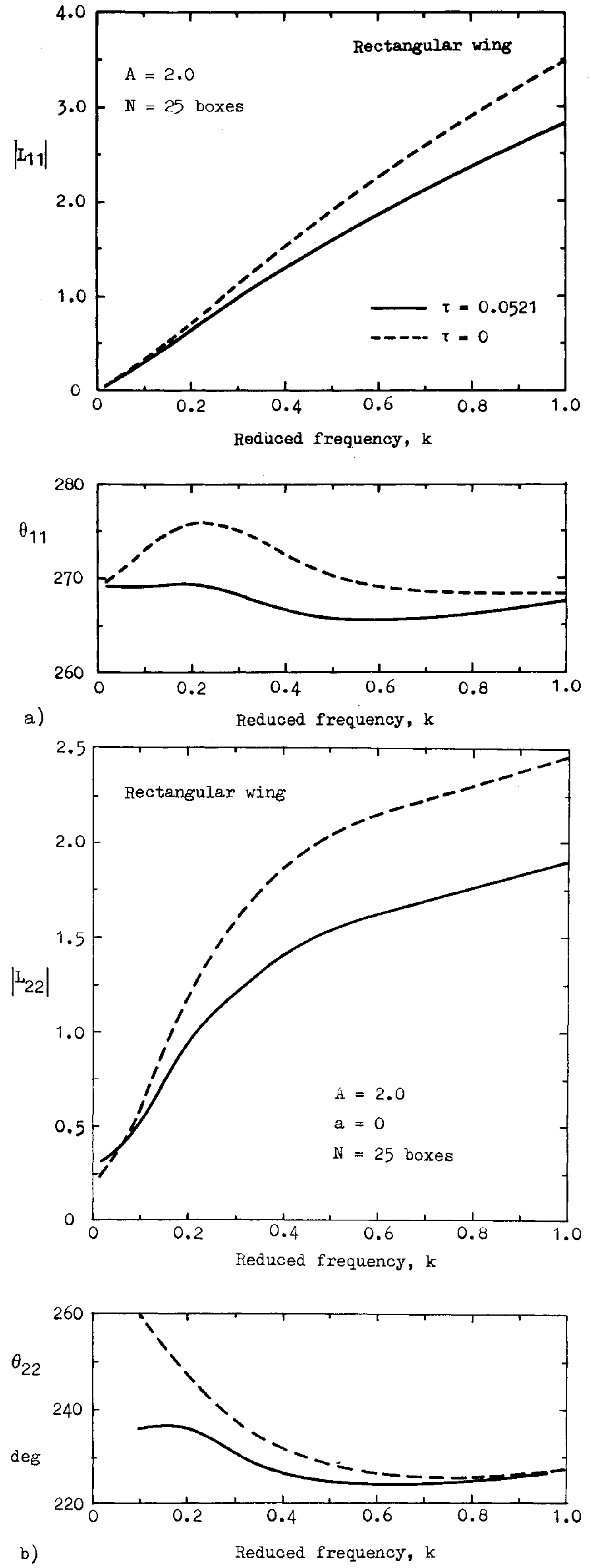

Fig. 4 Generalized aerodynamic forces for A = 2.0 rectangular wing at M_∞ = 1.0. a) Lift due to plunge; b) pitching moment due to pitch.

represented by a finite number of straightline segments and, based on these, the area is then approximated by a grid of square boxes. In each box, represented by its center, the flow parameters are considered to be constants

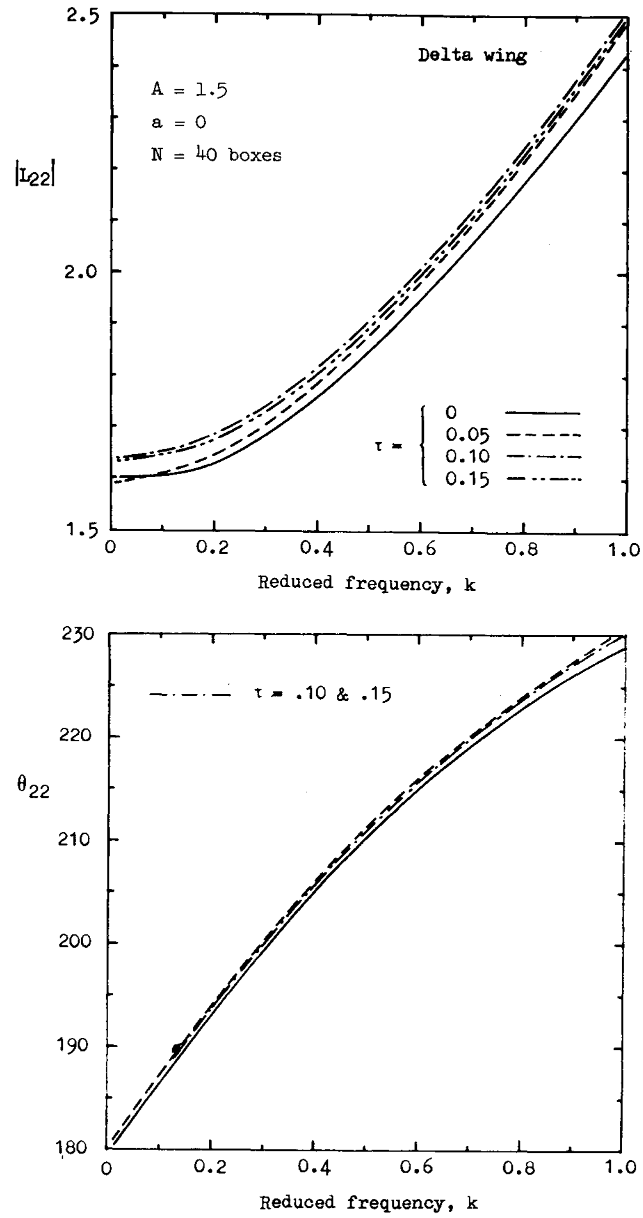

Fig. 5 Pitching moment due to pitch for A = 1.5 delta wing at M_∞ = 1.0

equal to their values at the box centers. The trailing edge of the wing must be unswept and there must be no control surfaces.

The doublet potential at the center of each box in the transformed space is obtained by using Eqs. (20) and (21), with a prescribed downwash distribution, from the sonic-box computer program. Then it is converted into the doublet potential of the physical wing by applying Eq. (11). That is, the doublet potential at the center of each box in the physical space is found by determining that of the corresponding point in the transformed space and applying the inverse transformation. This inversion results in a nonuniform array of points and boxes on the physical wing, but this method interpolates the data before further usage for integrations. The generalized aerodynamic force coefficients are computed from the following expression[2]:

$$L_{ij} = \frac{4}{S\delta_i} \iint_S (\varphi_{o,x} + ik\varphi_o)_i f_j \, dx \, dy$$

The stability derivatives are calculated by using the following relationships:

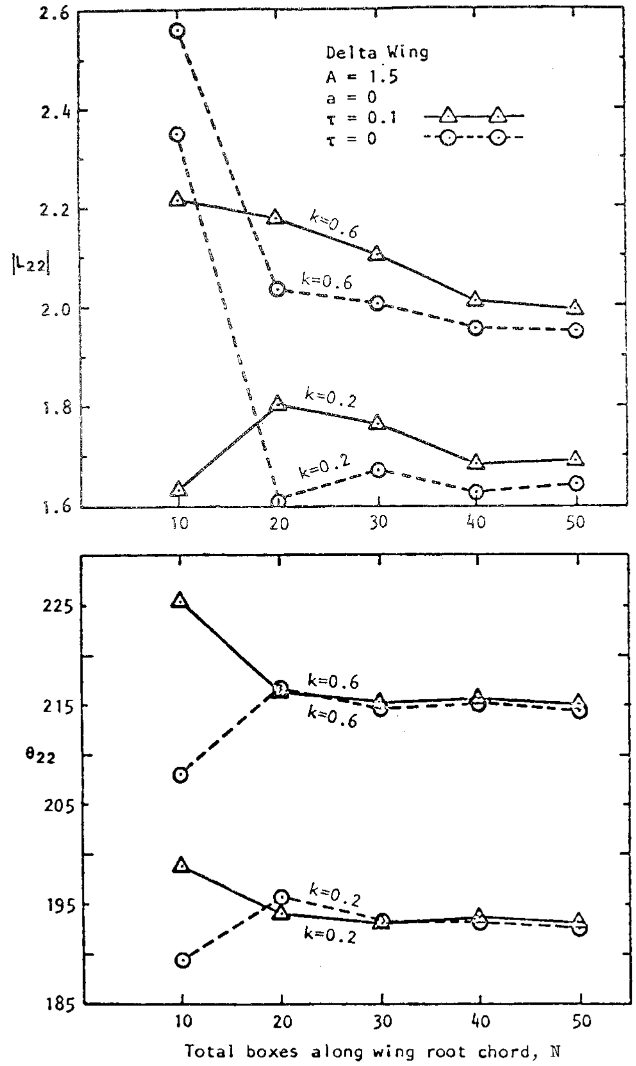

Fig. 6 Convergence of pitching moment due to pitch for A = 1.5 delta wing at M_∞ = 1.0

$$C_{L,\alpha} = -\frac{1}{k} Im[L_{11}]$$

$$C_{M,q} + C_{M,\dot{\alpha}} = \frac{1}{k} Im[L_{22} - a(L_{21} + L_{12}) + a^2 L_{11}]$$

V. Results and Discussion

Typical unsteady aerodynamic results have been calculated for rectangular and delta wings where the steady-state Mach number distributions are readily avaliable.[15] The rectangular wing of aspect ratio 2.0 has a biconvex airfoil section, and the delta wing for a variety of aspect ratios has an elliptic cross section in a plane perpendicular to the chordwise axis.

Sample results are shown in Figs. 4 and 5 for rectangular and delta wings, respectively. The rectangular wing is pitching about an axis along the leading edge; that is, a = 0.0. For the rectangular wing in sonic flow, Fig. 4 shows that the effect of 0.0521 thickness ratio is to reduce generalized force coefficients by as much as 20% compared with zero-thickness values. These changes are somewhat larger than anticipated, especially at the higher frequencies. Since the results have not yet been correlated with experimental data, they should be regarded as tentative. The steady Mach number distribution used in this calculation is quite rapidly varying, particularly near the leading edge of this wing,[15] so in the transformed coordinates the planform is severely distorted. The box size used may be too

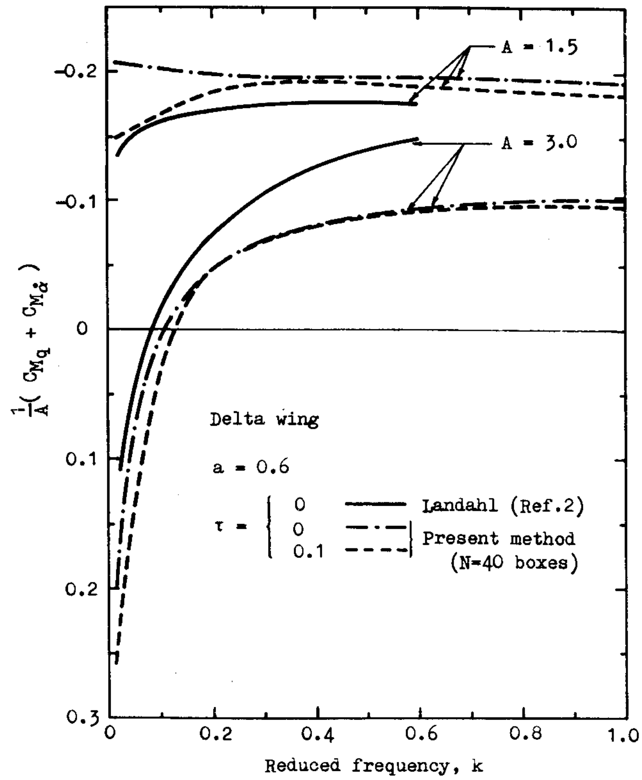

Fig. 7 Damping in pitch for delta wings pitching about a = 0.6 at M_∞ = 1.0.

large to define these variations adequately. Also, the requirement of the linearized theory that the reduced frequency satisfy the relationship $k \gg |\phi_x|$ is probably violated by the large velocity disturbances near the leading edge.

Variations of the generalized aerodynamic force coefficients for pitching moment due to pitch oscillations around the apex of a delta wing configuration with an aspect ratio A = 1.5 are shown in Fig. 5. A distribution of 40 boxes along the centerline chord was used in the computations. Figure 6 indicates that this number should be sufficient for reasonable convergence of the generalized force. (See also Figs. 17 and 20 of Ref. 6.) Pitching-moment magnitudes and phase angles show little change with thickness increase. At medium-to-high reduced frequencies, increases in absolute values are of the order of 5%, with the maximum occurring for 10% thickness. Over most of the frequency range, a further increase to 15% thickness results in a reversal of trend, or less increase in magnitude of the force coefficients.

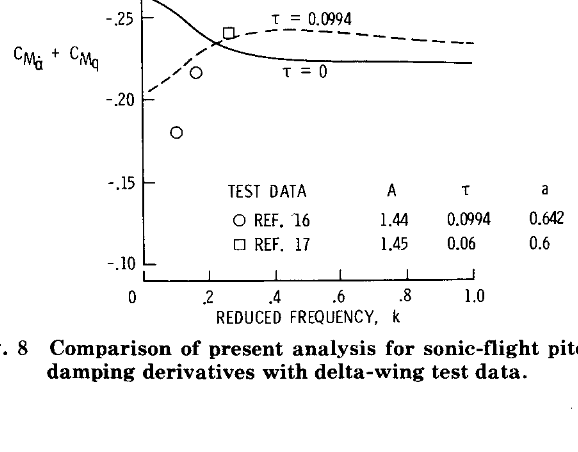

Fig. 8 Comparison of present analysis for sonic-flight pitch-damping derivatives with delta-wing test data.

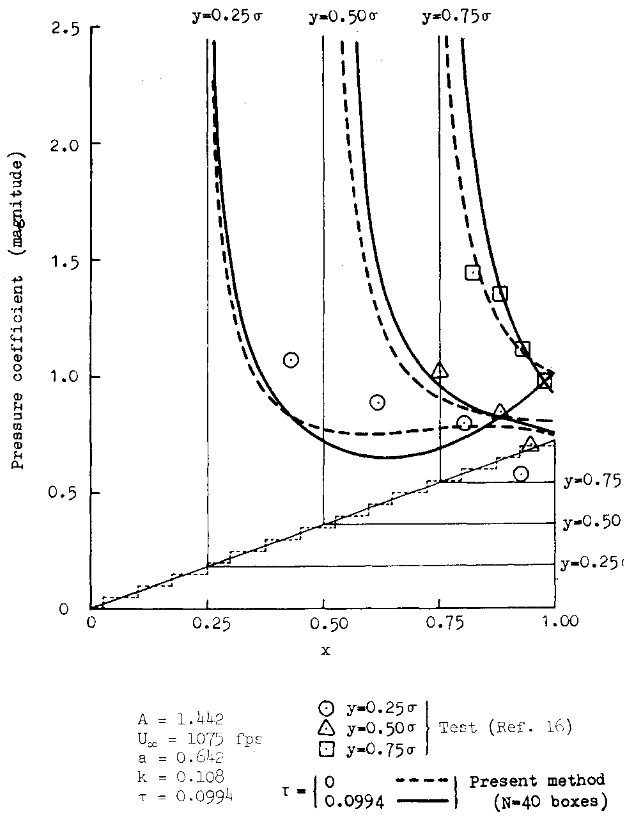

Fig. 9 Chordwise pressure variations at three spanwise stations on A = 1.442 delta wing pitching about a = 0.642.

Damping-in-pitch values for delta wings are compared with values for Landahl's well-known linearized theory[2] in Fig. 7 and with experiment[16,17] in Fig. 8. The effect of finite thickness on damping in pitch is indicated to be stabilizing and of appreciable magnitude at the lower reduced frequencies. At the higher frequencies, thickness appears to be of much less importance. The differences in damping magnitude (Fig. 7) obtained from the two zero-thickness methods (Landahl's and sonic-box), particularly for the higher aspect ratio and the higher frequencies, may be attributable, at least in part, to the upper-frequency and aspect-ratio limitations in Landahl's method. Figure 8 shows that accounting for finite thickness in the damping-in-pitch calculation produces correct trends in comparison with the limited experimental data.

Only a small amount of test data are available for oscillatory transonic flow over the present configurations. The data scatter is generally large, but the more reliable appearing data from Ref. 16 for a 70° delta wing oscillating at M_∞ = 1 are shown in Figs. 9 and 10 in comparison with the current calculations. The inclusion of thickness in the calculation of chordwise pressure distribution appears to improve agreement with experiment at the more outboard wing stations (Fig. 9). Data from the same tests[16] show good agreement with calculated values of lift-curve slope as a function of frequency. As expected, the effect of wing thickness on calculated lift-curve slope is small and is of the same order as the scatter in the test data. In comparison, appreciably greater effects of thickness on pitching-moment slope and aerodynamic center would be anticipated.

The only calculated results available for comparison with the rectangular and delta wing calculations in Fig. 10 were obtained from linearized theory (zero thickness). Landahl's theory[2] compares quite well for the rectangular wing at low reduced frequencies in Fig. 10a. Similarly, Davies' theory[3] agrees very well with results for the delta wing at aspect-ratio 1.5, shown in Fig. 10b.

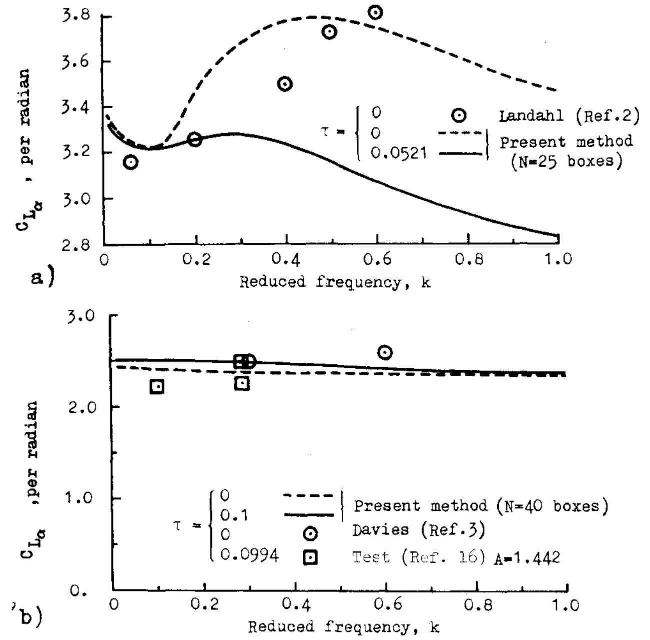

Fig. 10 Lift-curve slope variation with reduced frequency and thickness at M_∞ = 1.0. a) A = 2.0 rectangular wing; b) A = 1.5 delta wing.

The accuracy of the sonic-box method depends on the box size used in the computation,[6] and this is also true for the locally linearized sonic-box method proposed in this study (see Fig. 6). Some additional inaccuracy has resulted from the least-square surface-fitting technique used in the computation. Better surface-fitting techniques are available, however.[18] Because of the transformation technique adopted, certain types of wings for which a one-to-one transformation cannot be made must be excluded from treatment of the present approach. However, this difficulty may be alleviated by using a locally varying source strength distribution, related to the local Mach number, in place of the transformation presently employed.

Finally, generalized aerodynamic forces generated by the method developed herein have been used to assess the aerodynamic effect of finite wing thickness on the flutter characteristics of a 45° delta wing (A = 4.0) with elliptical cross section perpendicular to freestream direction; that is, an elliptic cone. The mass and stiffness of the wing were assumed to be contained entirely within a homogeneous plate of uniform thickness and same planform as

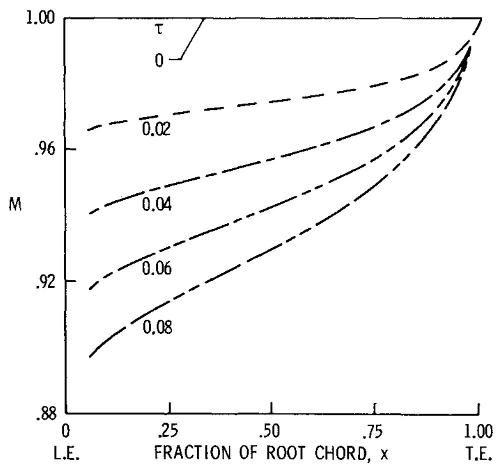

Fig. 11 Effect of thickness on local Mach number at M_∞ = 1.0 for A = 4.0 delta wing with elliptical cross section. Method of Ref. 15.

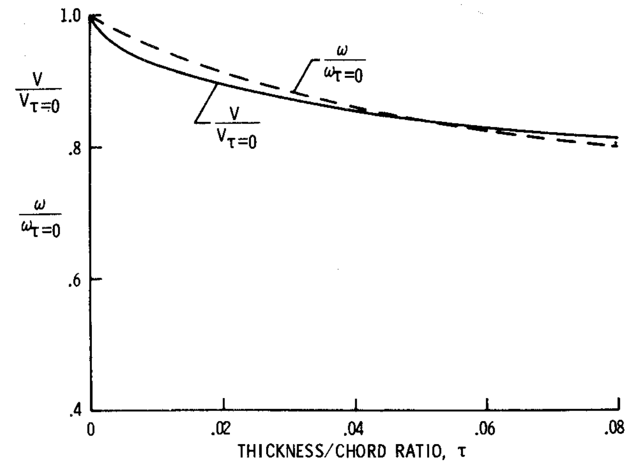

Fig. 12 Effect of thickness on flutter characteristics at $M_\infty = 1.0$ for $A = 4.0$ delta wing with elliptical cross section.

the wing. The wing was taken to be contilever mounted along its center line, and the first four natural undamped vibration modes were used in the flutter calculations. Four modes had previously been found to be sufficient to converge supersonic flutter calculations for the wing.

The distribution of local Mach number over the wing surface, which is required input for computation of the oscillatory aerodynamic loading, was calculated by the method of Ref. 15 and is shown in Fig. 11 for several values of thickness ratio. Note that for the elliptic cone, the method of Ref. 15 predicts no variation of Mach number in spanwise direction. No effect of base pressure was considered.

The effects of thickness on flutter speed and flutter frequency are shown in Fig. 12. It is seen that for a thickness ratio of 0.04, a realistic value for the present wing, flutter speed is predicted to be 15% lower than that predicted for zero thickness. Flutter frequency is also lower by a comparable amount.

VI. Conclusions

The local linearization concept is applied to the determination of approximate wing thickness effects in unsteady transonic flow using the sonic-box computational procedure. The principal argument in this concept is that the flow properties in a sufficiently small region on a wing may be treated as constant for calculation purposes. Thus, the small-perturbation velocity-potential equation with variable coefficients becomes locally linear. A coordinate transformation then reduces the equation to the well-known linearized transonic equation with constant coefficients, allowing the familiar sonic-box computer program to be applied in the transformed space.

Sample calculations for delta and rectangular wings are presented to demonstrate the capabilities of the program and to show the contribution of wing thickness effects to various unsteady-flow quantities. Comparisons are made with results from linearized theories as well as with limited available test data, and reasonable trends with thickness effects are indicated. Application to transonic flutter of a delta wing is also shown.

Although the method proposed in this study has not yet been thoroughly examined for a wide range of wings which satisfy the underlying assumptions of the present locally linearized sonic-box method, it is believed that the method should offer improvement over the zero-thickness linearized theory. The limitation caused by the transformation method can be improved by using a modified source strength distribution, related to the local Mach number, in place of the transformation presently employed to account for thickness effects. Also, improved accuracy should be obtained by replacing the least-square surface fitting procedure used in the original sonic-box computer program with updated techniques.

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